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Planetary orbits in binary stars

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Numerical integrations of the general three-body problem, with one component having a planetary mass, indicate that stable planetary orbits can exist in binary stars. The limitation for stability is that the ratio of the periastron distance of the outer tertiary component to the semimajor axis of the close component be somewhere in the range 3–4, regardless of which of the components is the planet. For most known binaries, this region of stability includes the region of habitability for planets.

INTRODUCTION

WITH the modern improvements in astrometric instrumentation and reduction procedures, there has been an increased interest in the possibility of detecting planetary companions to nearby stars by the technique of astrometric perturbations [see Gatewood (1976) for a recent discussion of problems and possibilities]. Many of the nearby systems, however, are double, which brings up the question of whether stable planetary orbits are possible in such systems, and if so, what are the dynamical limitations on the existence of such orbits. Indeed, multiple-star systems with more than two components are known to be stable, provided the arrangement is hierarchical (Harrington 1968, 1969); in these cases the motion is approximately two-body motion in the close binary, plus two-body motion of the tertiary and the binary barycenter. Further, the solar system is stable even if several of the planetary masses are augmented considerably over their present values (Nacozy 1967), and the existence of the extensive satellite systems of the outer planets indicates hierarchical dynamical systems are at least long lived. Thus, it is to be expected that stable planetary orbits are possible in binary stars, provided the planets are either sufficiently close to one of the components or sufficiently remote from the binary as a whole. The principal question, therefore, is that of the criteria or limitation for stability of such orbits.

Because a planet would have a very small mass compared to the stellar components, the results of the restricted three-body problem may provide some information. Indeed, the case of equal-mass primaries in circular orbits is the exhaustively studied Copenhagen problem, the hierarchical configurations being classes (f)–(i), (l), and (m) [see Szebehely (1967), Chap. 9, for a complete discussion of work on this problem]. Recognizing the importance of the binary eccentricity, Shelus and Kumar (1970) have studied various configurations of the elliptic restricted three-body problem, though still with fairly small eccentricities. All of these cases, however, have concentrated on periodic orbits and their linear stability, rather than orbital stability in more general cases. In addition, any (presumably limited) effects of the mass of the planet are neglected. A new alternative approach has been undertaken by Szebehely and his colleagues (e.g., Szebehely and Zare 1976; Szebehely and McKenzie 1977) in which they examine the conditions under which the zero-velocity surfaces open up. Their conditions, however, are, in a sense, limits only, but not sufficient ones at that, and in some cases give rather pessimistic results for the general three-body problem as compared to numerical results. Thus, it was decided to try to establish the stability criteria through direct experimentation via numerical integrations.

I. THE EXPERIMENT

A series of numerical integrations of general three-body systems, with one body much less massive than the other two, has been carried out to try to answer some of the questions just raised. Several characteristics of three-body motion, for the cases of comparable masses, have been assumed to apply to the present situation, in order to reduce the number of cases to be considered. First, it has been found (Harrington 1972) that motion in the third dimension (parallel to the total angular momentum vector) does not affect the stability characteristics, apart from the instability of the close binary if its plane of motion is perpendicular to the plane of motion of the tertiary. There is a bifurcation about this instability into the two cases of co- or counterrevolving binary and tertiary, but the actual inclinations are not significant. Hence, it was possible to consider only the two planar cases of prograde ($I = 0$) and retrograde ($I = \pi$) motion. Second, it was also found that the initial angles were not significant to the stability of the systems; hence, all integrations could be started with the systems at the same phase. For these experiments, the semimajor axes coincided in direction, the components of the close binary were at periastron, and the tertiary was at apastron, thus minimizing the perturbative influences in the initial motions. Third, it was found that stability depended only logarithmically on the masses of the large bodies (Harrington 1975), and therefore it varied only moderately over the mass range found for most stars;

TABLE I. Lower limit of a_2/a_1.

	$I = 0$		$I = \Pi$	
$m_p^{-1}\backslash e_B$	0.5	0.0	0.5	0.0
Planet close to one component				
1000	7	4	7	3
300 000	7	4	7	4
Planet outside binary				
1000	4	2	3	3
300 000	4	2	3	2

hence, only cases in which the massive bodies had equal mass were considered.

The various permutations of cases integrated were as follows:

(a) The stellar binary had an eccentricity of 0.0 or 0.5 (approximating known spectroscopic and visual systems, respectively).

(b) The planet circled one component of the binary or the entire binary, but in both cases in an initially circular orbit.

(c) The planet had a mass, compared to the mass of each of the stellar components, of 1000^{-1} (approximately Jupiter) or $300\,000^{-1}$ (approximately Earth). The semimajor axis of the close pair was normalized to unity, and that of the tertiary about the binary was varied in increments of unity until a minimum value for stability was found. Stability is defined here as it has been in the previously cited publications by this author. That is, stability means bounded motion only, in particular, that the semimajor axes and eccentricities show no secular or large periodic variations over the rather limited time span covered by the simulations. The actual judgment of stability was made solely by examination of the axes and eccentricities and, hence, has a certain degree of subjectivity associated with it. Further, because of the way the initial conditions were chosen, the stated limits can be regarded as sufficient conditions for stability only in a probabilistic sense.

II. RESULTS

The derived lower limits on the axes ratios (a_2/a_1) for stability for the various cases mentioned are given in Table I. The limit is in the range 3–4, except for two general cases. First, when the massive tertiary is in an eccentric orbit, the lower limit is significantly greater. This is consistent with the previous results (Harrington 1972, 1975), in which it was found that the periastron distance of the tertiary (hence, the ratio q_2/a_1) is the significant parameter. For the cases examined here, this ratio is still within the range 3–4. Second, when the close, equal-mass binary has a circular orbit, the limit is somewhat lower. This also is not unexpected, since the increased symmetry of equal masses plus circular orbit might well increase the stability of the system. In any case, it is not inconsistent with a limit in the range of 3–4 as a sufficient condition for stability. In conclusion, even for the planetary case, the critical parameter for stability appears to be the ratio q_2/a_1, and it is probably in the range 3.5–4.0 for corevolving systems and 3.0–3.5 for counterrevolving systems if the massive components are of equal mass.

These results can be combined with the previous results on the stability of the general three-body problem to produce empirical functional stability criteria. An earlier suggestion for this functional form has been made (Harrington 1975) which obviously does not apply to the case of one component of very small mass. The following condition for stability of a three-body system is now suggested:

$$q_2/a_1 \geq F \equiv A\left\{1 + B\log\left[\frac{1 + m_3/(m_1+m_2)}{3/2}\right]\right\} + K.$$

The masses of the primary, secondary, and tertiary are given by m_1, m_2, and m_3, the parameters A and B are to be determined empirically, separately for co- and counterrevolving cases, and K is 0 if this is to be a mean fit, and is approximately 2 if it is to be an upper limit. The coefficient A is the limit on q_2/a_1 for the equal-mass case and is taken directly from those results (Harrington 1972); B is then determined by a least-squares fit to the unequal-mass cases (Harrington 1975). The results are given below:

I	A	B
0	3.50	0.70
Π	2.75	0.64

The extensions of these fits to the planetary case are given in Figs. 1 and 2, in which the results from the various studies are plotted with the above fits. Note that this analysis has attempted to establish criteria for the geometry of the systems, given certain values for the masses of the components. The work of Nacozy (1976), for example, would establish additional constraints as functions of the masses.

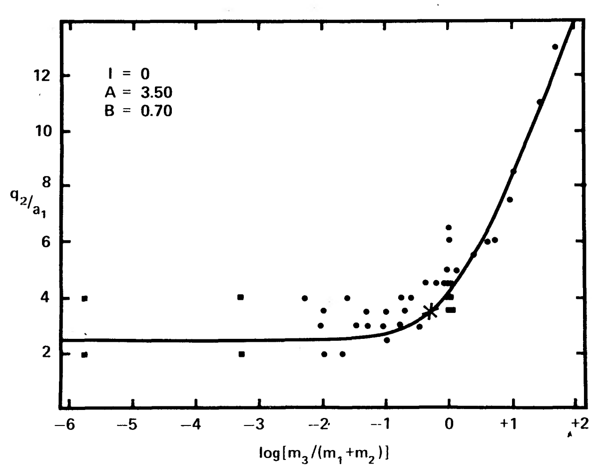

FIG. 1. Limit on q_2/a_1, for stability as a function of the ratio of the mass of the tertiary to the mass of the binary; corevolving cases. The asterisk marks the equal-mass case, the dots the results from the unequal- but comparable-mass cases, and the squares the results from the planetary cases. Also shown is the mean fit.

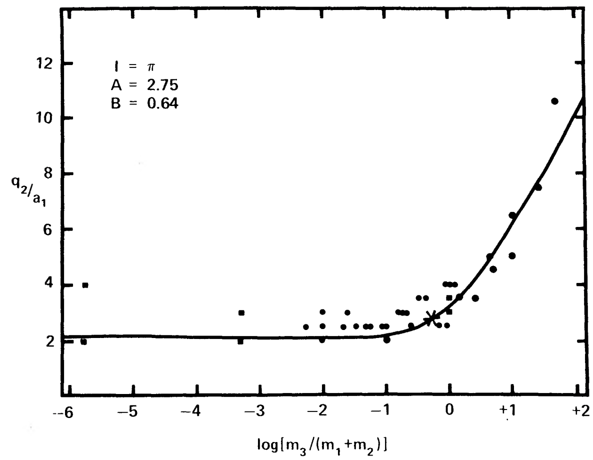

FIG. 2. Same as Fig. 1; counterrevolving cases.

III. THE SOLAR SYSTEM

As part of a separate effort to develop a solar system integration procedure, several numerical integrations of the present system were carried out, with suitable modification to introduce the effects of a binary star. The initial conditions and masses employed were those given by Oesterwinter and Cohen (1972). For testing and comparison, the present solar system was integrated for the same 50-yr period covered by Oesterwinter and Cohen, and the agreement with the final elements (a, published to ten significant figures, and e, published to seven significant figures) was perfect, with the exception of Mercury (due to the omission of relativistic terms here), and Neptune (agreement to only 1 part in 10^6, for unknown reasons).

Two special integrations were then run. First, the Sun was replaced by a binary with components each of mass 0.5, semimajor axis 0.2, and eccentricity 0.5. The binary was fixed, in that the perturbations of the planets on the binary were neglected, making it possible to make the modifications only in the calculations of the forces. In this case, Mercury ($q_2/a_1 = 1.54$) escaped from the system almost immediately, going to $a = 0.86$, $e = 0.66$ in 100 days, and to $a = -3.4$, $e = 1.1$ in 4300 days. The other planets showed virtually no additional perturbations (for Venus, $q_2/a_1 = 3.59$), the only detectable effect being a slightly greater range of variation in the eccentricities.

The second simulation increased the mass of Jupiter to $1 M_\odot$. The initial velocities of the outer planets were increased by the factor $[2/(1 + m_{\rm Jupiter})]^{1/2}$ in order to keep the same initial elements. Because of the short interval covered by these integrations, only the inner planets were examined. The three innermost planets showed no significant additional perturbations (for the Earth, in this case, $q_2/a_1 = 4.95$). Mars ($q_2/a_1 = 3.25$) immediately showed large perturbations, with its heliocientric distance varying between 0.5 and 4.3 and its eccentricity sometimes approaching 0.9. It did not escape in the time covered, but its motion would certainly be classified as unstable in the sense employed here. Thus, these experiments with a modified solar system generate results consistent with those from a consideration of the general three-body problem.

Ten-year temperature curves were calculated for the Earth for each of the above situations, assuming black-body behavior for the Earth; the results are given in Fig. 3, with temperatures normalized to 300 K at 1 AU from $1 M_\odot$. Figure 3(a) is for the present solar system, Fig. 3(b) is for the binary Sun, and Fig. 3(c) is for the solar Jupiter. It can be seen that temperature variations on the Earth would not be significantly greater in case (b) or (c) than they are in case (a).

IV. HABITABILITY

For some considerations, a second question may have to be considered—that of whether the planet is habitable. For a first analysis, habitability will be defined only as the planet being in a stable orbit such that (a) the eccentricity is close to zero and (b) the time-average total luminosity received at the planet is close to unity when measured in terms of solar luminosity. Since apparent luminosity depends on intrinsic luminosity L and distance r as L/r^2, and the time average of $1/r^2$ is $1/\eta a^2$ [$\eta = (1 - e^2)^{1/2}$], the condition on luminosity for habitability can be stated as follows:

$$\Sigma\ (L/\eta a^2) = 1.$$

Here, the summation is over all luminous bodies, and the elements should be of the planetary orbit with respect to each of the luminous bodies.

The above condition can be combined with the previous condition for stability to formulate a single general condition for habitability of a planet in a binary star that depends only on observable parameters of the binary orbit and the components. Let a, e, and η refer to the

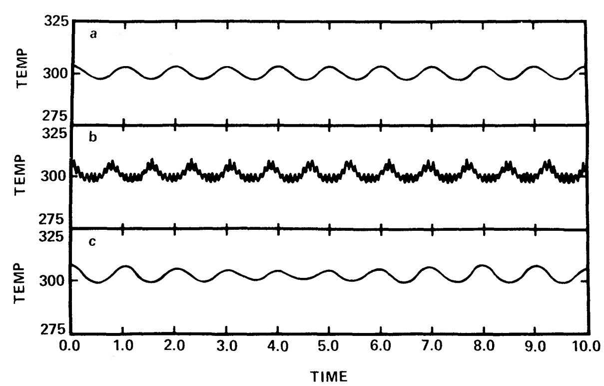

FIG. 3. Mean temperature on the Earth, normalized to 300 K at 1 AU from $1 L_\odot$. Time is in years from arbitrary starting point. Part (a), present solar system. Part (b), Sun replaced by close binary. Part (c), Jupiter augmented to $1 M_\odot$.

TABLE II. Known nearby binaries.

System	F_{I1}^2	F_{I2}^2	F_0^2
α Cent	80	290	0.00
L726-8	6×10^4	9×10^4	0.00
Sirius	2.3	···	0.06
Procyon	9.9	···	0.00
70 Oph	230	1230	0.00
Krug 60	2×10^4	4×10^5	0.00
O^2Eri	···	6×10^5	0.00

orbit of the binary, a_p,e_p (=0) refer to the orbit of the planet about either component or the binary barycenter (all semimajor axes in astronomical units), and subscripts 1 and 2 refer to the primary and secondary stellar components. Two cases must be considered. The first case is when the planet is orbiting close to the primary (the case of orbiting the secondary is the same, with reversed numerical subscripts). Therefore, we have $a_1 = a_p$, $a_2 = a$, $a(1-e)/a_p \geq F$, and $L_1/a_p{}^2 + L_2/\eta a^2 = 1$. Eliminating a_p from the above, the following first condition for a binary which could have a habitable planet results:

$$(\eta a^2 - L_2)(1 - e^2)/L_1 \geq F^2.$$

The second case is when the planet is orbiting the binary. In this case we have $a_1 = a_2 \simeq a_p$, $q_p \simeq a_p$, $(L_1 + L_2)/a_p{}^2 = 1$, and $a_p/a \geq F$. Again eliminating a_p, we have the following second condition for the possibility of having a habitable planet:

$$(L_1 + L_2)/a^2 \geq F^2.$$

If good astrometry and photometry are available for a system, L_1, L_2, a, and e can be estimated reliably. Rougher estimates can be obtained by assuming luminosities, and hence distance, from spectral type or color, and zero eccentricity and inclination for the binary orbit (hence $a = r$). In either case, a reasonable estimate of whether a habitable planet could exist in a binary star is possible, and if so, whether it would orbit just one of the components or the entire system.

The habitability conditions have been calculated for a few nearby visual binaries, these being the systems most likely to reveal the existence of a planet through astrometric perturbations. The results are tabulated in Table II, where the criteria have been calculated for a planet orbiting the entire system F_0^2, or either component, F_{I1}^2 or F_{I2}^2(except white dwarf components). In all cases, habitable planets are not possible around the entire system, since the systems are themselves quite wide. However, since the theoretical limiting value of F is generally on the order of 10, it appears from these considerations that most systems could carry habitable planets close to one component (there may, of course, be cosmological reasons why such systems would be unlikely). The only possible exceptions are Procyon, which is borderline, and Sirius, for which a habitable planet, by this definition, seems clearly impossible.

V. CONCLUSION

It has been found that it is, indeed, possible to have stable planetary orbits in binary stars, provided the planet closely orbits one of the components, or it orbits the binary at a distance. Further, the regions of stability of such orbits are sufficiently large that they may very well include the zones of habitability for most binaries. Therefore, binary stars should not be excluded as candidates for objects with planets, even habitable planets.

REFERENCES

Gatewood, G. (1976). Icarus **27**, 1.
Harrington, R. S. (1968). Astron. J. **73**, 190.
Harrington, R. S. (1969). Celest. Mech. **1**, 200.
Harrington, R. S. (1972). Celest. Mech. **6**, 322.
Harrington, R. S. (1975). Astron. J. **80**, 1081.
Nacozy, P. E. (1976). Astron. J. **81**, 787.
Oesterwinter, C., and Cohen, C. J. (1972). Celest. Mech. **5**, 317.
Shelus, P. J., and Kumar, S. S. (1970). Astron. J. **75**, 315.
Szebehely, V. (1967). *Theory of Orbits* (Academic, New York).
Szebehely, V., and McKenzie, R. (1977). Astron. J. **82**, 79.
Szebehely, V., and Zare, K. (1977). Astron. Astrophys. **58**, 145.